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Comprehensive Rotorcraft Analysis Methods

Wendell B. Stephens
2GCHAS Project Engineer
Aeroflightdynamics Directorate
U.S. Army Research and Technology Activity - AVSCOM
Ames Research Center, Moffett Field, CA 94035-1099
and
Edward E. Austin
Aerodynamics/Dynamics Team Leader
Aviation Applied Technology Directorate
U.S. Army Research and Technology Activity - AVSCOM
Ft. Eustis, VA 23604-5577

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Abstract

The paper describes the development and application of comprehensive rotorcraft analysis methods in the field of rotorcraft technology. These large scale analyses and the resulting computer programs are intended to treat the complex aeromechanical phenomena that describe the behavior of rotorcraft. They may be used to predict rotor aerodynamics, acoustics, performance, stability and control, handling qualities, loads and vibrations, structures, dynamics, and aeroelastic stability characteristics for a variety of applications including research, preliminary and detail design, and evaluation and treatment of field problems. The principal comprehensive methods developed or under development in recent years and generally available to the rotorcraft community because of US Army Aviation Research and Technology Activity (ARTA) sponsorship of all or part of the software systems are the Rotorcraft Flight Simulation (C81), Dynamic System Coupler (DYSCO), Coupled Rotor/Airframe Vibration Analysis Program (SIMVIB), Comprehensive Analytical Model of Rotorcraft Aerodynamics and Dynamics (CAMRAD), General Rotorcraft Aeromechanical Stability Program (GRASP), and Second Generation Comprehensive Helicopter Analysis System (2GCHAS).

1 Introduction

For the past decade the Army Aviation Research and Technology Activity (ARTA) has sponsored programs directed toward the development of comprehensive rotorcraft analysis methods. The goals in sponsoring this development have been twofold. The first goal has been to provide a basis from which to advance the state-of-the-art in analysis methods. The second goal has been to provide the Government with analytical tools which could be used to evaluate present and proposed Army rotorcraft accurately, quickly and fairly. These analytical methods developed have also provided the rotorcraft manufacturers with tools which have been used in their own design efforts.

The rotorcraft industry matured and the vehicle proved to be vital to the Army's missions just as mainframe computers became available. Rotorcraft analysts made use of this new computational capability to solve numerically the complex sets of nonlinear differential equations that describe the aerodynamics and dynamics of rotorcraft. From specialized analyses formulated and programmed by one or two individuals there evolved more comprehensive analyses which merged two or more disciplines and which became important factors in unifying the design process (ref. [1]). However, Ormiston (ref. [2]) showed that the best industry analytical tools provided wide differences in the predicted loads for a hypothetical simple rotor system. The Army's UTTAS and AAH competitions exposed serious difficulties in modeling the elastic coupling in flexbeam rotor systems and in assessing the influence of main rotor wake on the fuselage and the tail. These problems were intimately related to inaccurate predictions in the technical disciplines of loads, vibrations, performance, stability and control, and aeroelastic stability. Schrage (ref. [3]) illustrated the problem that confronted the Army during the recent Advanced Helicopter Improvement Program (AHIP) Source Selection Evaluation Board (SSEB). The AHIP SSEB. The AHIP board used 27 different codes to accomplish this proposal evaluation. The analysis software had to be verified on similar data bases in advance of the board and available on computers to be used during the evaluation. The input and data bases for these programs were basically incompatible with one another and each program required a trained expert knowledgeable in its use. Thus, the inaccuracies and the incompatibilities of software available has spurred both Army and industry interest in improving the analysis methodology.

In order to address the need for better analytical tools for rotorcraft, the ARTA began focusing on near term and long range solutions to the multidiscipline analysis problems. In the near term, a current industry code was enhanced and encouraged for use as an industry standard. This was the C81 code (ref. [4]) developed by Bell Helicopter Textron. The primary funding for this enhancement came from the ARTA Aviation Applied Technology Directorate (AATD). Further, to establish an improved capability for analyzing designs for

low vibration rotorcraft and include the effects of rotor/airframe coupling, the development of the new SIMVIB code (refs. [5,6]) by Sikorsky Aircraft Corporation was supported by the ARTA Aerostructures Directorate (ASD). This particular code was to be used in support of an ongoing Army-NASA rotorcraft validation program effort.

To address the longer range problem of obtaining an advanced state-of-the-art code which would provide an industry-wide accepted standard for analysis, two other initiatives were undertaken. The AATD supported a set of three pre-designs for an interdisciplinary computer system that would address a broad range of technical disciplines. This interdisciplinary concept for the pre-design effort was referred to as the Second Generation Comprehensive Helicopter Analysis System (2GCHAS). The pre-designs were summarized in references [7,8,9]. The other initiative was an inhouse effort by NASA and the ARTA's Aeroflight-dynamics Directorate (AFDD) which resulted in the CAMRAD program (refs. [10,11,12]). This effort was referred to as a "generation and a-half" capability since it was more mathematically consistent and comprehensive than the available industry codes but still fell short of the ambitious 2GCHAS requirements.

The state of the rotorcraft analysis capability was thoroughly reviewed at the conclusion of the 2GCHAS pre-design activity at a workshop sponsored by the AFDD (ref. [13]). A review of the Army options at that time spawned three new efforts. First, the major long range goal of developing 2GCHAS was re-affirmed and the task for development of the code was assigned to the AFDD. Second, this effort was augmented by supporting a prototype code which addressed an approach to satisfying the dynamic coupling requirements for 2GCHAS using concepts introduced by Kaman Aerospace Corporation during the 2GCHAS pre-design (ref. [8]). The features addressed by the DYSCO code were basically related to structural dynamic modeling and little attention was given to aerodynamic modeling (ref. [14]). Third, an inhouse research effort to advance the state-of-the-art was initiated at the AFDD. This study was directed toward improvements in modeling elastic structures by developing a higher-order beam element capable of undergoing large elastic deformations, and implementing kinematic constraints capable of unrestricted rotations. The code developed from this research effort was referred to as GRASP (ref. [15]). Although the technical discipline capability in GRASP was limited to hover stability problems, the beam element and multi-body connectivity concepts were far in advance of what was being used in finite element codes and it was anticipated that these concepts would be included in future versions of 2GCHAS. Even though, strictly speaking, the GRASP code is not a multidisciplinary code, its influence on multidisciplinary code future development provides the justification for including it in the discussion of this paper.

The multidisciplinary codes developed by the ARTA sponsorship fall into three development categories. The industry-developed codes are C81, DYSCO,

and SIMVIB. The Government-developed codes are CAMRAD and GRASP. The 2GCHAS code occupies a unique category of joint industry-Government development. In the following sections the codes will be described briefly and compared with respect to function, math basis, and applications. Other codes have been surveyed in reference [16,17].

2 Program Descriptions

2.1 Rotorcraft Flight Simulation, C81

The Rotorcraft Flight Simulation code developed by Bell Helicopter Textron is best known by its program designation C81 (ref. [4]). It is nearly as old as the modern era in mainframe computers. It was originally programmed for the IBM Model 7070 computer as a rotor model to provide rotor inputs to an aircraft stability and control analysis. It has evolved through a combination of Government and Bell IR&D development funds and its use in SSEBs was an important factor as it became the most widely distributed first generation comprehensive analysis. C81 provides an interesting study in program evolution. As the original stability and control oriented program was given the capability for time varying maneuvers it was broken in to parts - a primary processing program and a post-processor to do plotting and analysis of time history data. When an aeroelastic rotor analysis was added, a rotor blade eigenanalysis was required to provide rotor blade natural frequencies and mode shapes, but it ran as a separate step. Recently, an aircraft design optimization feature has been wrapped around C81 and an executive has been added to control the various optional analyses in the system. It is this new program that will be discussed through the remainder of the paper.

The executive developed to accomplish this is designated the Rotorcraft Design Optimization Computer Program (RDOCP) and the suite of programs under its control include: (1) an input parsing and control program, (2) the system FORTRAN compiler, (3) the system linker; (4) the Myklestad Rotor Natural Frequency Program (rotor blade eigenanalysis), the C81 primary analysis and either of two commercially available nonlinear programming programs integrated into a single job step, (5) a Myklestad plotter, (6) the post-processor program for C81, (7) a C81 plotter, and (8) a DATAMAP interface program (ref. [18]). A schematic of the RDOCP executive is shown in figure 1. The input parse and control program sorts the input stream into files for each of these programs as required for the requested mode of operation and creates job control language for subsequent job steps. The inclusion of the system FORTRAN compiler and linker permit routine definition of nonstandard objective and constraint functions for the design optimization problem in FORTRAN as well as

temporary modification of the C81 code. The program has eight modes of operation: (1) Myklestad only, (2) C81 only, (3) Myklestad followed by C81, (4) Optimized Myklestad, (5) Optimized C81, (6) Myklestad followed by optimized C81, (7) Optimized Myklestad followed by C81, and (8) Optimized Myklestad and C81 together. While many operational enhancements have been made to C81 and program bugs have been eliminated, there have been no major technical discipline improvements to the primary C81 code in the last ten years.

In one of the more complex scenarios, the executive would enable the user to modify the C81 FORTRAN source code, create in FORTRAN his own objective and constraint functions for an optimization problem, relink the object modules, define the initial aircraft parameters using a previously defined database and NAMELIST changes, calculate rotor blade eigenvalues and eigenvectors, compute an aircraft trim condition using the complete set of governing nonlinear differential equations, and optimize the design as described by any of the continuous input variables. In a subsequent run the user could analyze the response of this design in a maneuver, interrupting the maneuver to determine stability derivatives at any moment using a perturbation technique, and plot any of several thousand available outputs on the line printer or a plotter. The user could save the output on a file and use the postprocessing code, DATAMAP, for additional analysis and graphical display.

The analysis is applicable to rotorcraft with up to two rotors in all common configurations. Rotor blade and pylon dynamics are represented using the modal method. Single load path teetering, gimballed, hingeless and articulated rotors may be modeled directly. Multiple load path rotors may be approximated in the Myklestad analysis by inputting hub mass and stiffness properties developed using NASTRAN, by using specially developed subroutines for specific bearingless configurations, or by adapting well-documented hub component subroutines to account for unique concepts. Pylon modes must also be calculated using an external analysis such as NASTRAN. The primary analysis capabilities are rotor and aircraft performance, flight path dynamics, airframe stability and control, and rotor loads in trimmed and maneuvering flight. Rotor aeroelastic stability may be inferred by simulation of an appropriate maneuver to excite the mode of interest and post-processing the response time history.

Rotor aerodynamic analysis options include 2-D airfoil coefficients from tables or curve fits; two methods to generate unsteady corrections, 3-D corrections, and modified Glauert inflow or harmonic coefficient inputs based on external data or analysis. Trim is obtained using a modified Newton-Raphson technique including either first harmonic elastic response or direct numerical integration of the system differential equations. In addition to the basic force/moment/one-per-rev flapping trim, other trim options include constant power climbs and descents, steady pullups and turns, rotor only and a number of others.

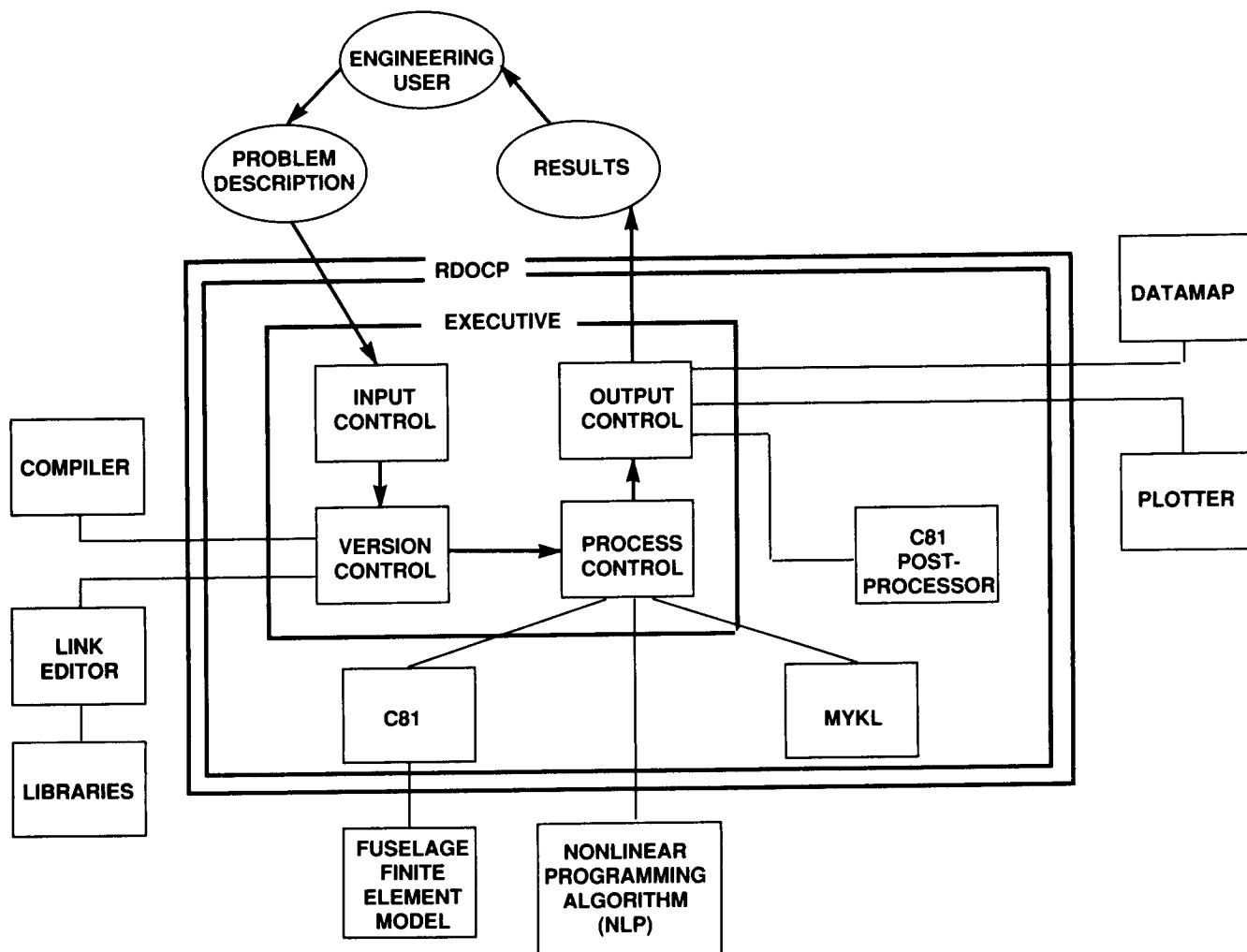


Figure 1: C81 modularity and command relationship.

Aircraft and analysis components are described using individual data groups which may be stored in a data base and include: airfoils as generalized curves or tables, rotors, rotor modes, pylon/fuselage modes, nonuniform induced velocity distribution, rotor induced velocities at lifting surfaces, fuselage, wing, stabilizing surfaces, jets, external stores, dive brakes, control rigging, bobweight, weapons and SCAS. Transient maneuvers which may be simulated include: control displacements from trim positions; rotor tilting; vertical, horizontal and vortex induced gusts; engine power changes; auxiliary thrust; weapon fire; changing RPM, rotor brake; SCAS failures; deployment of drag brakes and stores; and changing incidence angles of lifting surfaces. In addition there is a simplified pilot representation which attempts to maintain aircraft trim attitude upon experiencing these transient effects and a capability to follow a trajectory specified in terms of angular rates and normal load factor.

2.2 Dynamic System Coupler, DYSCO

In contrast to programs like C81 which grew as engineering analyses and subsequently had executive features added, DYSCO was initially developed as an executive concept demonstration which was almost totally devoid of useful technical analytical capabilities. The concept demonstrated by Kaman Aerospace Corporation (refs. [19,20]) was that transformation matrices similar to those proposed by Hurty, (reference [21]) could be used to assemble a set of rotorcraft system differential equations from a number of sets of subsystem differential equations by comparing the names the program user had assigned to the subsystems' degrees of freedom. In DYSCO, assigning the same arbitrary name to degrees of freedom in two subsystems couples the two systems together at those degrees of freedom, e.g., giving the name FRED to the lateral hub degree of freedom for a rotor and to the mast lateral degree of freedom for the fuselage equates those two components in the lateral direction (and the lateral direction only). Improved rotor modules and input facilities were added to the program after the initial demonstration was completed satisfactorily (ref. [22]). Experience with the resulting code indicated that DYSCO was well suited to the investigation of nonstandard airframe configurations such as those due to damage and that the addition of a rotor blade damage model would provide a comprehensive and easy-to-use damage assessment technology. Under an ongoing contract, the AATD is sponsoring development of the blade damage model, the incorporation of a global reference system and several other features. In addition, the Air Force sponsored the development of capabilities for applications not related to helicopters. Added capabilities are general modal representation of three-dimensional structures, a landing gear model, enhanced solution techniques, general forces, linear constraints and an improved eigenanalysis to include nonproportional damping terms. Demonstrations of these new capabil-

ities will be provided at the Air Force Wright Aeronautical Laboratories and the AATD in the spring of 1987.

The version of DYSCO discussed herein is DYSCO 4.0. The basic operational scenario for DYSCO is a three-step procedure which consists of defining the subsystems (or components), defining the system (or model), and executing a solution. The modularity of the DYSCO architecture is illustrated in figure 2. The components available in DYSCO include a modal, thin-beam fuselage, a rotor with rigid blades, a rotor with elastic blades, a rotor control system with elastic rods and swashplate, a general structural finite element, a nonlinear spring, and an arbitrary linear constraint. Force modules include a sinusoidal shaker, two-dimensional aerodynamics from empirical equations or tables, a more general rotor aerodynamics module, fuselage flat plate drag, and fuselage linear aerodynamics. The more complex rotor and fuselage aerodynamic modules are based primarily on the same technology as that contained in C81 and do not represent an improvement in the state-of-the-art.

As shown in figure 2, the solution module, "define solution" is separated functionally from the "form coupled system" math model formation. The solution modules available in DYSCO include eigenanalysis using the power method and the Householder method, a time history solution using Runge-Kutta, a Floquet stability analysis, a trim analysis using the technique of periodic shooting, and a frequency domain mobility calculation. The rotor, hub and airframe forces and responses are fully coupled and the system equations of motion are solved simultaneously.

The primary capability of DYSCO is to calculate dynamic response and loads, from which some overall performance predictions can be obtained. Until the global coordinate system is added, which will permit direct representation of gravitational effects, DYSCO will not be suitable for aircraft flight path dynamics predictions and there are no plans to extend the system to the areas of stability and control or to acoustics.

2.3 Coupled Rotor/Airframe Vibration Analysis Program, SIMVIB

The concept for SIMVIB, as shown in figure 3 and described in reference [5], was to develop an interactive coupled rotor/airframe analysis package which could be used for preliminary design of new rotorcraft or for vibration treatments of existing aircraft with a emphasis on providing good interactive response. To do this, on-line computing requirements were kept low by doing the real number crunching computations off-line with a suite of programs provided with the base program. The effect of the fuselage on rotor aerodynamics, the rotor trim

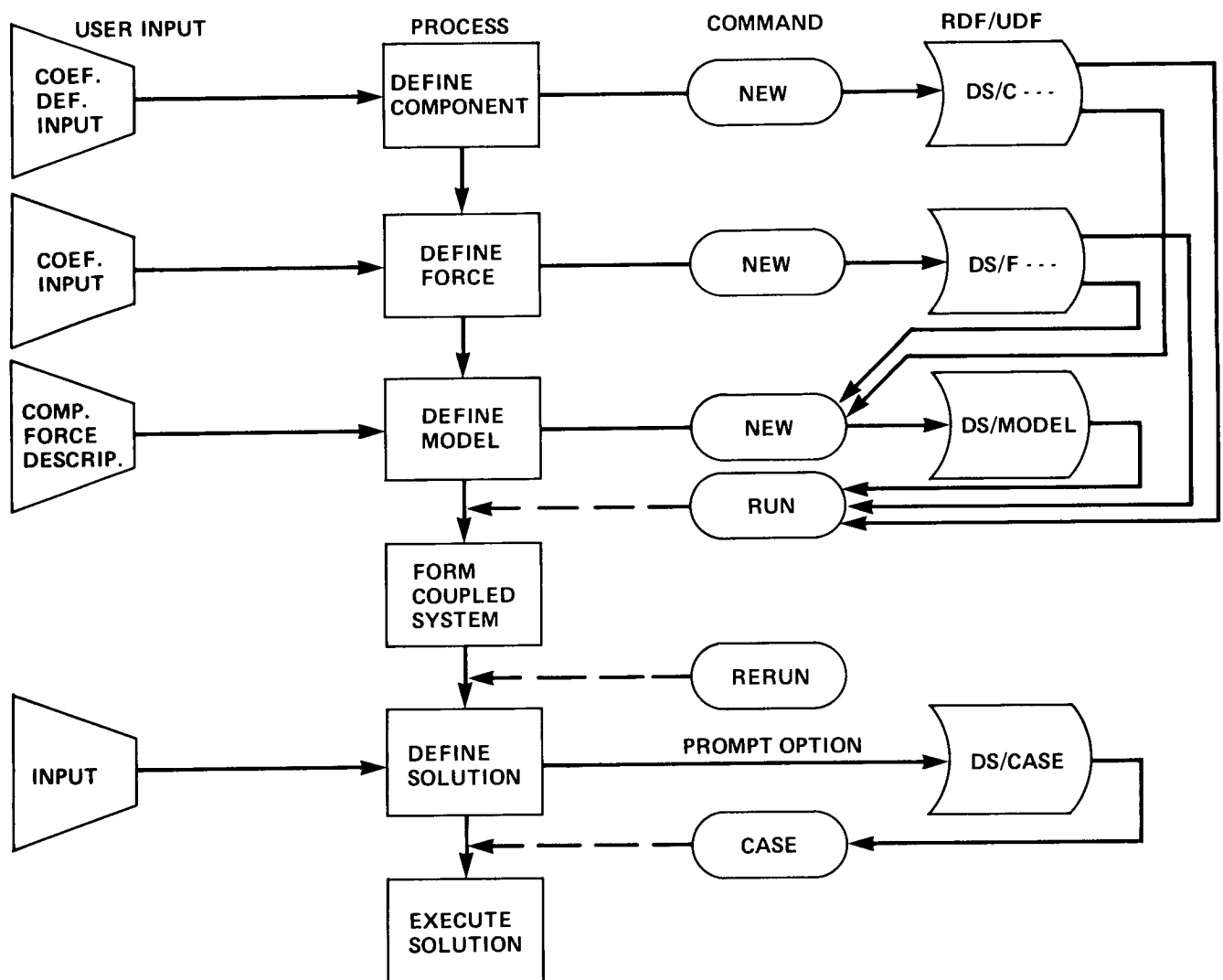


Figure 2: DYSCO modularity and command relationship.

state, rotor forces, rotor impedance, and fuselage vibratory forces as affected by the rotor are all calculated externally to the base program. The base program couples subcomponents using the Hurty method and provides steady forced response, time-varying response and eigenvalue solutions. Although a suite of programs was provided to perform the external calculations, any other analysis can be used if data formats are consistent with SIMVIB.

Figure 3 illustrates the data transfers between the base program and the suite of external programs. Iteration between programs is indicated by two separate arrows between the programs and the labels on the arrows characterize the data being transferred. The data transfers are accomplished by file transfers and the base program operates in a stand-alone mode to obtain solutions.

Substructures available in the base program include a vibration absorber, a generalized force, a uniform elastic beam, a connection constraint, an aeroelastic rotor model expressed in terms of mass, damping and stiffness matrices (for hover only), a modal representation of a dynamical structure, in-plane and out-of-plane bifilar absorbers, an anti-resonant isolator, and an impedance model of an aeroelastic rotor for hover or forward flight. Higher harmonic control is modeled by an externally derived transfer matrix relating hub forces and moments to swashplate inputs.

2.4 Comprehensive Analytical Model of Rotorcraft Aerodynamics and Dynamics, CAMRAD

The CAMRAD program was developed in the late 1970's (refs. [10,11,12]). Its purpose was to provide a computationally reliable and efficient multidisciplinary dynamic and aerodynamic analysis for the design, testing and evaluation of rotors and rotorcraft. An excellent overview of the program and its applications is found in reference [23]. The analysis is applicable to general two-rotor aircraft configurations. The rotor systems allowed include articulated, hingeless, gimbaled, and teetering rotors with an arbitrary number of blades. The rotor configurations may be single main rotor, main rotor - tail rotor, side-by-side, tandem or tilting proprotor. The analysis capabilities extend to rotor performance, loads, and noise; helicopter vibration and gust response; flight dynamics and handling qualities; and system aeroelastic stability. The math model also includes a drive train set of equations which accounts for the engine, governor, shaft flexibility, and rotor rotational speed degrees of freedom.

The analysis capability is outlined in figure 4. The solution process begins with determining a trim state in which the rotor and airframe motion are periodic and the controls for a specified flight condition are calculated. The program allows the user to select various trim options based on the usual six

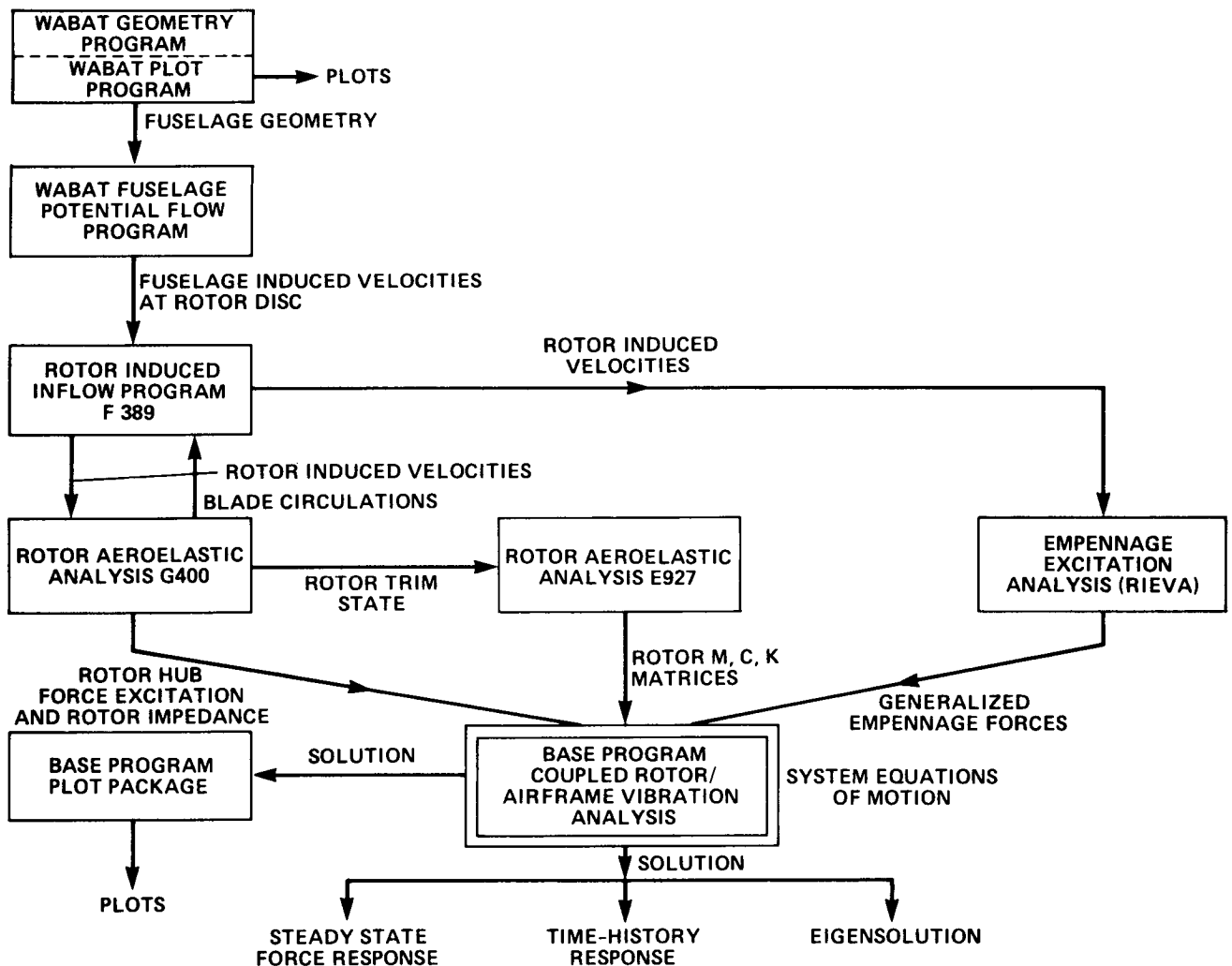


Figure 3: SIMVIB data flow diagram.

force and moment equations and an additional power available equation. There are 26 preset trim options in the code representing flight and wind tunnel trim conditions.

The transient and flight dynamics solutions are based on quasi-static rotor solutions. The trim, transient and flight dynamics solution procedures use a common rotor analysis. The solution of the equations is separated into two parts based on the assumption that the aircraft motion is quasi-static when compared with the rotor speed. This assumption allows the periodic rotor motion to be used for the transient motions of the helicopter as well as the trim motion. By taking advantage of the frequency separation of the rotor and aircraft motions, an economical solution procedure is obtained. One part, therefore, is the periodic, harmonic solution of the rotor and airframe vibration. The other part is the time domain solution airframe motion including the aircraft rigid body, rotor speed perturbations, and static elastic deflection of the airframe and drive train.

In the transient solution the rigid body equations of motion are numerically integrated for prescribed gusts or control inputs to calculate the nonequilibrium flight path. In the flight dynamics solution, perturbation of the body motion and controls are calculated yielding time invariant linear differential equations for the aircraft rigid body motions. The poles, zeros, and eigenvectors define the aircraft flying qualities.

Output from the trim, transient and flight dynamics solutions can be processed to obtain specific technical discipline output in performance, loads, vibration or noise prediction.

The flutter analysis constructs a linear set of differential equations for all variables of the aircraft and rotor(s) in order to define the system stability. The equations may be time invariant as for the axial flow flight conditions or time variant having periodic coefficients. In the latter case, a Floquet solution is obtained. Additional capabilities allows the periodic coefficients to be averaged and quasi-static reductions to be made if desired by the user.

The rotor aerodynamics is based on two-dimensional steady airfoil characteristics with corrections for three dimensional and unsteady flow effects, including dynamic stall. Three options of inflow calculation are allowed: uniform inflow, nonuniform inflow with prescribed-wake geometry and nonuniform inflow with free-wake geometry. The uniform inflow is based on an empirical model using momentum theory and includes a linear variation over the rotor disk. The rotor wake model is based on vortex lattice approximations of the wake and wake influence coefficients are calculated for incompressible flow. Rotor/rotor interference is accounted for as is interference velocities at the airframe. Wake rollup and distortion effects are included in the model.

STEADY STATE FLIGHT
CONDITION

SOLVE FOR CONTROLS
AND PERIODIC MOTION

NUMERICAL INTEGRATION
OF TRANSIENT RESPONSE
(QUASISTATIC ROTOR)

STABILITY DERIVATIVE
CALCULATION (QUASI-
STATIC ROTOR)

ANALYSIS OF TIME-
INVARIANT LINEAR
DIFFERENTIAL
EQUATIONS

LINEAR DIFFERENTIAL
EQUATIONS

ANALYSIS (TIME-
INVARIANT OR PERIODIC)

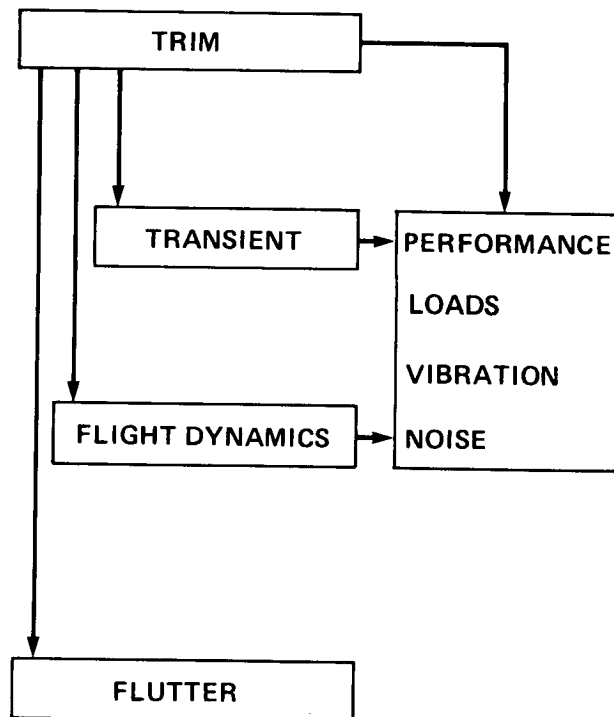


Figure 4: CAMRAD tasks and solutions.

2.5 General Rotorcraft Aeromechanical Stability Program, GRASP

The GRASP program (ref. [15]) is an in-house code developed by the AAFD. It is an outgrowth of the FLAIR code (ref. [24]), and the problems associated with modeling a bearingless rotor system and elastic blades. The GRASP code was developed to apply advanced modeling and finite-element techniques to complex hub and blade behavior. In particular, blade/root kinematic modeling has been enhanced greatly by GRASP. The GRASP code is described as a hybrid between finite element codes and spacecraft-oriented multi-body programs. Using this combination, the coupling constraints at the blade root are general and can account for large rotations, time dependency, and nonlinearities. The substructuring in GRASP is extensive and allows the user to build up complex structures from a reasonably small library of elements and constraints. The main structural element in GRASP is the aeroelastic beam which is an elastic, kinematically-nonlinear, variable-order beam element subject to inertial, gravitational and aerodynamic loads. The equations of motion are not explicitly derived but are calculated inside the GRASP code. Although using small strain theory assumptions, the code development does not make any small angle approximations or use an ordering scheme to truncate terms. The analysis capabilities apply only to static nonlinear and linearized dynamic behavior. The program does not handle periodic terms and is therefore restricted to axial flight regimes. The stability problem is described as an asymmetric eigenproblem where the mass matrix is symmetric but the damping and stiffness matrices may be asymmetric. The mass matrix includes apparent mass from the air. A simple air mass model using induced inflow dynamics with uniform axial freestream velocity is used in GRASP.

In order to allow the modeling flexibility required, GRASP has an extensive executive capability referred to as an "information manager" to control the execution sequence and to manage the data structures. The information manager selects the dimension size required for each data structure during a run and efficiently manages the data.

2.6 Second Generation Comprehensive Helicopter Analysis System, 2GCHAS

The 2GCHAS is a Government-sponsored project which had its origin in the mid-1970's. It was a response to a need by the Army for a more comprehensive analysis tool to be used by helicopter designers in the industry and aircraft evaluators in the Army. The basic concepts for the system were formulated in a series of competitive predesign studies (refs. [7,8,9]). The predesign studies

were initiated to define the requirements of the System from the both technology discipline and user-interface standpoints. These requirements were later summarized in reference [25]. The System is divided into two complexes; the Executive Complex and the Technology Complex. A contract was awarded to Computer Sciences Corporation to develop the Executive Complex prior to developing the Technology Complex. The Executive Complex precedes the Technology Complex by approximately two years in development. The completed 2GCHAS is expected to be available to the public in the third quarter of 1988 after System integration of the CPCI deliveries and as shown by the schedule chart in figure 5. The public release of the system is referred to as first level release (FLR). This release will only be for the VAX VMS operating system. It is the intent of ARTA to continue the code development and support over a period of time so that advanced technology modules can be added to the System and to allow the System to be converted to other operating systems.

The Executive Complex is that part of the System which controls the execution of the technology analyses, supports the run-time data management, provides a user interface for input and output, has available a database management system for I/O storage, and provides utilities for graphic and printed output of analysis runs. The development of the Executive Complex is accomplished in five builds, each of which is of approximately 6 months duration; the fourth build was delivered to the Project Office in February 1987 and the final build will be delivered in September 1987. The third build, which was delivered in September 1986, has been installed at the Technology Complex contractor sites for use in developing the technology software. The project office will upgrade the technology contractor deliveries after acceptance testing of the succeeding builds are complete. The schedule for the 2GCHAS development is shown in figure 5. The Executive Complex line in the figure depicts the time periods for the five builds.

The Technology Complex is that part of the System which provides the technical capability to perform particular interdisciplinary engineering analyses of rotorcraft such as performance, loads and vibration, aerodynamics, stability and controls, and aeroelasticity stability. This Complex has been divided into six contractual units called computer program configuration items (CPCIs). Four CPCIs were awarded in January of 1986 and two were awarded in January of 1987. All six contracts will be completed in the second quarter of 1988. The finite element library will be developed McDonnell Douglas Helicopter Company, the hierarchical assembly procedure and the maneuver and trim solution processes will be developed by Kaman Aerospace Corporation, the linear analysis and eigensolutions will be developed by Advanced Rotorcraft Technology, Incorporated, the aerodynamic capability will be developed by Boeing Vertol Company, the technology discipline output processing will be provided by Sikorsky Aircraft Corporation, and input processing and coordination capability will

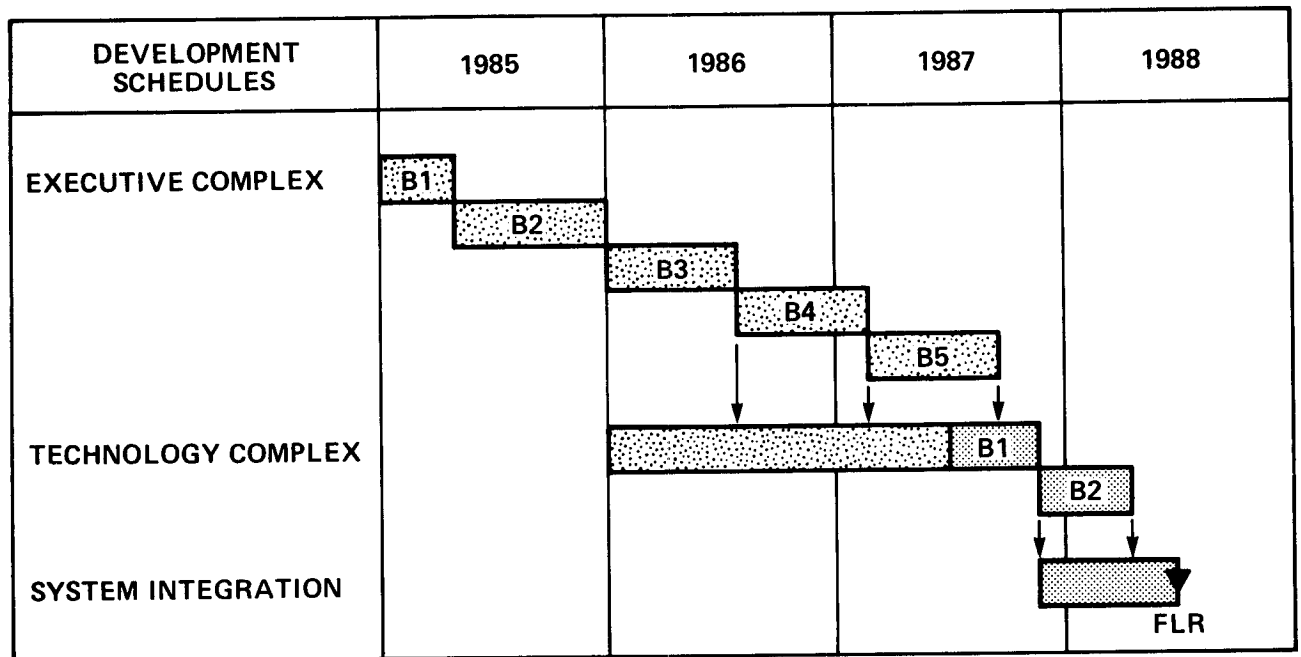


Figure 5: 2GCHAS schedule chart.

be provided by Sterling Software Incorporated. Most of the CPCIs' software delivery will be based on a two-build schedule as shown in figure 5.

The technology basis of the Technology Complex is a finite element, time domain assembly process with solution algorithms available for maneuver, periodic response, trim and eigenanalysis. The eigenanalysis extends to both constant and periodic coefficient equations. The assembly process is hierarchical with provision for modal synthesis, multipoint constraints, coordinate system transformations, singlepoint constraints, and multiblade coordinates. The aerodynamic computational capability includes the induced velocities and airload distributions. The induced velocities are calculated from momentum theory with vortex-element, prescribed-wake methods in both axial and forward flight regimes. The airloads are based on the lifting line strip theory approach. The solutions to maneuver, trim or eigenvalue equations of motion are postprocessed to provide the appropriate engineering discipline results in such areas as loads, performance, stability, etc.

The analysis flexibility of 2GCHAS is depicted in figure 6. The Technology Complex operations move from left to right beginning with the user model building activities. The fundamental calculation to be performed is the trim solution. Once the trim state has been established, the user may proceed to the transient (maneuver) calculation, perturb the equations of motion or go directly to the last column to obtain the desired engineering discipline data

from postprocessing the solution state. If the perturbed equations are used, the user may perform either an eigenanalysis or a transient analysis of the linearized equations. Again, after solving the equations, the user selects the appropriate engineering discipline output form for the data and displays the results on the screen, plotter, printer or external file. The Executive Complex assists the user in identifying the path through the various solution algorithms via a user language; allows the user to store, recall, restart or checkpoint data along the path; aids the input preparation processes; provides utilities for the graphic interface; manages the data and data structures during the runtime; and provides diagnostic and status information during the run. The figure depicts the executive compiling a user command in a sequential fashion.

3 Comparisons of Capabilities

For the most part, the programs discussed in this paper attempt to be "comprehensive" in function. This comprehensiveness can be interpreted in various ways, however. A program can be comprehensive by addressing each of the technical disciplines indicated in table 1. The program might also be deemed comprehensive if its modeling capability is general such as that found in finite element codes. Such a code would allow the math models to range from very simple to quite detailed thereby allowing the models generated to be applicable to preliminary design, detailed design and research studies. Comprehensive codes require executive services support which involve data base management, user language features, graphics interfaces, and efficient modular execution of program segments. The tables presented in this section attempt to address comprehensiveness from all of these standpoints.

In table 1 the codes are compared with respect to technical disciplines. The asterisk indicates the capability is presently in the code. The exception is the 2GCHAS code where the capabilities are not yet present but are under contract. Only the CAMRAD directly produces results in each discipline. Even the 2GCHAS code has deferred the acoustic predictive capability from the first set of its contracts. The C81 code has recently been enhanced to include an optimization capability. The SIMVIB base program is strictly a vibration reduction design tool. However, the other analyses in the package provide capabilities in the areas of loads, aeroelastic stability and performance. GRASP is restricted to stability applications and axial flight regimes. The DYSCO code has emphasized the solution to the governing sets of equations and has not been tailored to the specific engineering disciplines. The DYSCO code solves for both trim and transient response.

In table 2 the executive features of the codes are compared. The presence

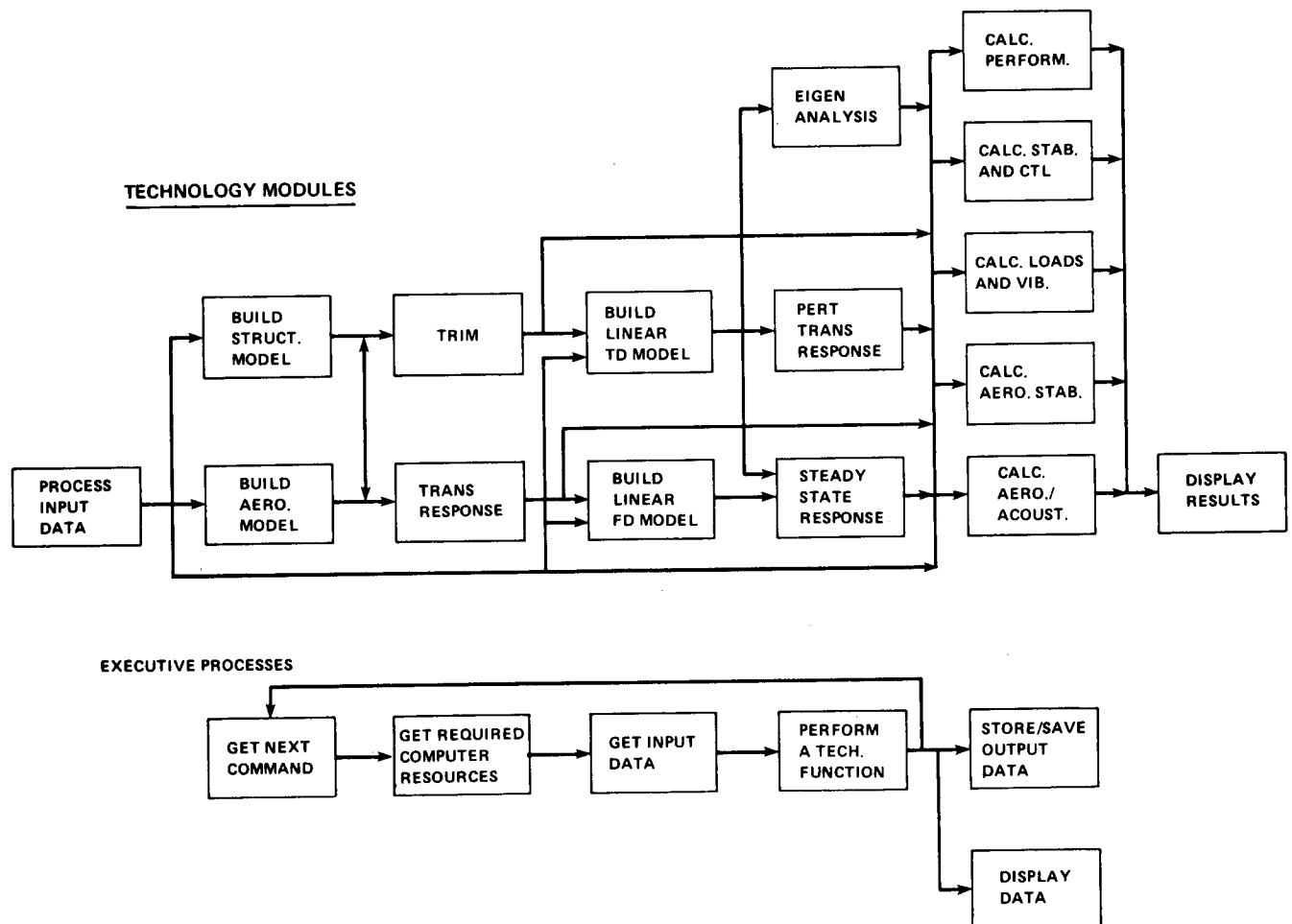


Figure 6: 2GCHAS Executive and Technology Complex functions.

Table 1: Comparison of Analysis Capabilities

Code	Per- form- ance	Loads and Vib.	Aero- elastic Stab.	Stab. and Control	Aero- dynam.	Other
C81	*	*	*	*	AF	Opt
CAMRAD	*	*	*	*	AF	Acst
DYSCO	*	*	*		AF	Dis
SIMVIB	*	*	*		AF	
GRASP		*	*		A	
2GCHAS	*	*	*	*	AF	

A = Axial flight, F = Forward flight, * = capability present,
Opt = optimization, Acst = acoustics prediction, Dis = dissimilar blades
allowed

of the features is an indicator of ease of use and services available to user during an analysis run or set of runs. Two approaches have been used. Codes such as SIMVIB and C81 have attempted to make maximum use out of existing software to perform segments of the overall analysis. SIMVIB is described in figure 3 and couples the various wake geometry, inflow dynamics and modal blades codes together via external files. C81 relies on external codes to generate blade modes, provide graphic output and perform optimization tasks. The C81 code has automated the program coupling procedures by providing assistance to the user in recompilations and JCL declarations. The C81 graphics capability is provided by its own specialised plotting programs and by an interface with the DATAMAP package (ref. [18]). The remaining codes attempt to have all technical capabilities internal to their system. The CAMRAD program has an input preprocessor and has few other executive services. Its graphics capability is restricted to a line printer two dimensional printouts. The user must provide his own interface to an existing graphics programs. The DYSCO program provides an extensive input processor and an environment wherein elements, analyses, and output can be easily added to the system. A database exists to the extent that restarts and data modifications can be easily performed. A graphics capability is not currently a part of the system. The GRASP code has a sophisticated information manager built into the system which automates the dimension space needed from user inputs. It also provides services to the user in input preparation. It uses an internal program language to set procedures for analysis executions. Graphics are not included in the system. The 2GCHAS code has the most extensive executive. The executive provides a robust runtime database which can be saved, edited, and restarted. The executive traps all program crashes and provides an environment within 2GCHAS for input prepa-

Table 2: Comparison of Executive Features

Code	Executive	Data Base	Graphics	Coupled System
C81	Auto JCL	File I/O	Ext/Int	Programs
CAMRAD	Preprocessor	File I/O	Some	*
DYSCO	File R/W	File I/O		*
SIMVIB	File Sys	File I/O		Programs
GRASP	Info. Mgr.	Runtime	External	*
2GCHAS	User Lang/Procs	Runtime	Internal	*

* = capability present

ration, graphics, and output control. The executive provides a user language to control the runtime execution. Alternatively, the user may write procedures in the user language or modules in FORTRAN which can be executed within the 2GCHAS environment. The graphics capability is linked directly to the DI-3000 commercial software package.

In table 3 the modeling capabilities of the programs are compared. All programs exhibit the ability to model a wide range of the current rotor types and at two least rotors within the model. The finite element-based codes (DYSCO, SIMVIB, GRASP, and 2GCHAS) have the potential for modeling an arbitrary number of rotors, even though the practical interest of one or two rotors is sufficient. In addition, the finite element codes are capable or have the potential, given an appropriate element library, of modeling a single blade as well as an arbitrary number of blades within the rotor system. The C81 and CAMRAD codes have a limit on the number of blade modes which can be used; the C81 code can include up to 11 modes and the CAMRAD code can include 10 bending modes and 5 torsion modes. In all cases, the airframe component can be elastic although greater model detail is accounted for in the finite element model. Other aircraft components can be modeled as required by finite element codes and the CAMRAD code provides the capability to model engine and transmission systems dynamics. Redundant load paths can be accounted for in the finite element codes provided the beam element properly accounts for axial extension. The fixed model programs (C81 and CAMRAD) provide the best aerodynamic modeling capabilities. In terms of wake geometry modeling and accounting for aerodynamic interferences these programs' capabilities are more mature and well matched to the analyses. The external program coupling of SIMVIB also provides a similar level of aerodynamic modeling. The 2GCHAS will be the first finite element code which attempts to emulate this level of aerodynamic modeling. Surprisingly, only CAMRAD and SIMVIB currently tie the nonlinear inflow calculation to a wake geometry and induced velocity calculation iteration.

Table 3: Comparison of Mathematical Model Features

Analysis	C81	CAMRAD	DYSCO	SIMVIB	GRASP	2GCHAS
Rotor Types	abghst	aghst	abghst	abghst	abghst	abghst
No of Rotors	2	2	4	*	*	*
No of blades	≥ 2	≥ 2	≥ 1	≥ 1	≥ 3	≥ 1
Blade Modes	11	10/5	5/5/3	FE	FE	FE
Elast Airfrm	Modal	Modal	FE/mod	FE	FE	FE
Components		Eng/Trns	FE/mod	FE	FE	FE
Redund Ld	approx		FE	FE	FE	FE
Inflow Dyn		*		*	*	*
Aero Inter	*	*				*
Free Wake		*				
Non inflow		*		*		*

a = articulated, b = bearingless, g = gimballed, h = hingeless, s = semiarticulated, t = teetering, FE = finite element, * = capability present, mod = modal, 5/5/3 = flap/lag/torsion modes, 10/5 = bending/torsion modes

In table 4 the structural modeling building capabilities are summarized. The modal analysis capability is not internal to all codes. The C81 code uses a specially developed Myklestad code for developing its modes and only DYSCO requires the user to provide his own modes. A multiblade coordinate transformation is provided by those codes which perform an aeroelastic stability analysis in forward flight. Model building using a hierarchical tree is allowed to some degree in DYSCO and to a general level in GRASP and 2GCHAS. General kinematic constraints which couple the elements or components may be nonlinear in only the GRASP code. Limited nonlinear coupling is accounted for in the CAMRAD program. It will be necessary for the other finite element codes to adopt a Lagrange multiplier scheme which will allow nonlinear coupling. The GRASP provides a screw coupling capability that accounts for very large displacements or rotations similar to those which occur in robotics where multi-body mechanisms must be accounted for. Single point constraints or displacement boundary conditions are provided by all the programs.

4 Published Results

The purpose of this section is to provide some insight into how a particular code has performed when compared to experimental results. In most cases only one

Table 4: Comparison of Transformations Allowed

Transform	C81	CAMRAD	DYSCO	SIMVIB	GRASP	2GCHAS
Modal	see a	Int.	Ext.	G400	Int.	Int.
MBC	*	*			*	*
Tree Sub			Some	Some	*	*
Nonlinear		Some			*	
SPC	Some	*	*	*	*	*

* = capability present, a = DYNAM06

result from the literature for each code is presented. An in-depth look at how the codes have performed in the area of predicting rotor loads has been provided in reference [26].

4.1 C81 Correlation

The AATD sponsored three efforts to determine the validity of C81. The reference [27] effort was conducted by Bell Helicopter using AH-1G data and the 1976 version of the program. Since the present rotor analysis in C81 is nearly identical to the 1976 version, and since the Myklestad analysis at that time was valid for teetering rotors, the results shown in figure 7 are representative of the current program. The flight condition represented in the figure is for a forward velocity of 129 KTAS using data counter 615 of the 8319 pound AH-1G aircraft. The analysis results show that the steady moments do not compare well at the inboard stations with the flight data. The oscillatory or peak-to-peak loads, however, are in much better agreement. Both the measured and calculated results are dominated by the one-per-rev frequency content. For the one-per-rev case the outboard comparisons are poor. The agreement in the higher-per-rev distributions is much better.

References [28] and [29] document earlier correlation efforts conducted by contractors other than the program developer. Both efforts were performed using the 1974 version of the program which was the starting point for the 1976 improved program. Reference [28] describes serious shortcomings in the program and concludes that accuracy for predictions of H-53 and S-67 helicopter characteristics did not exceed that of other analyses which cost less to run. Reference [29] noted good correlation for trim and performance for the Messerschmitt-Boelkw-Blohm BO-105 helicopter, reasonable correlation of main rotor flap bending moments, and poor correlation of main rotor chord

and shaft bending moments. Difficulties were experienced in attempting correlation of the stability and control characteristics of the BO-105. Some of the difficulties reported in these references may be due to inexperience of the user, some on limitations of the Myklestad analysis, and some on the C81 program itself. AATD has recently sponsored an effort to improve the Myklestad analysis for hingeless and articulated rotors. Later versions of C81 have been used by the Bell Helicopter for in-house correlations with articulated and hingeless configurations with better results than those presented in these references.

4.2 DYSCO Correlation

As discussed above, a current technology rotor analysis has only recently been added to DYSCO. The recent enhancement also included a correlation effort which was limited to rotorcraft performance characteristics. Some of the results are shown in figure 8. The only other published correlation was a comparison of ground resonance results using the Floquet stability analysis with the analytical results of Hammond (reference [30] for isotropic and nonisotropic hub characteristics. The results of the study (ref. [31]) showed exact agreement for all cases.

The results shown in figure 8 are taken from reference [22] where comparisons are made with operational loads survey test flight data (ref. [32]). Three DYSCO models are used in the comparison. The label, AH-1G, in figure 8 refers to the flight vehicle and the 36 refers to the data counter number in the flight test. The letter (S) in the label refers the DYSCO model with steady aerodynamics; the letter (U) refers to the DYSCO model with unsteady aerodynamics included; and the letters (B1S) refers to the DYSCO model with rigid out-of-plane mode and steady aerodynamics. The results for collective control, fuselage pitch and horsepower are generally good while the cyclic sine and cosine predictions are poor.

4.3 SIMVIB Correlation

Published comparisons of SIMVIB predictions with test data are limited to references [5] and [33]. Reference [5] includes correlations with a one-sixth scale model rotor system tested in the NASA Langley Research Center Transonic Dynamics Tunnel with and without higher harmonic control inputs as shown in figures 9-(a), 9-(b), 9-(c). Agreement with fuselage accelerations, variation of blade moments with advance ratio, and radial distribution of bending moments is good. The prediction of bifilar mass motions shown in figure 9-(d) is excellent, but it should be noted that rotor forces were inferred from bifilar base

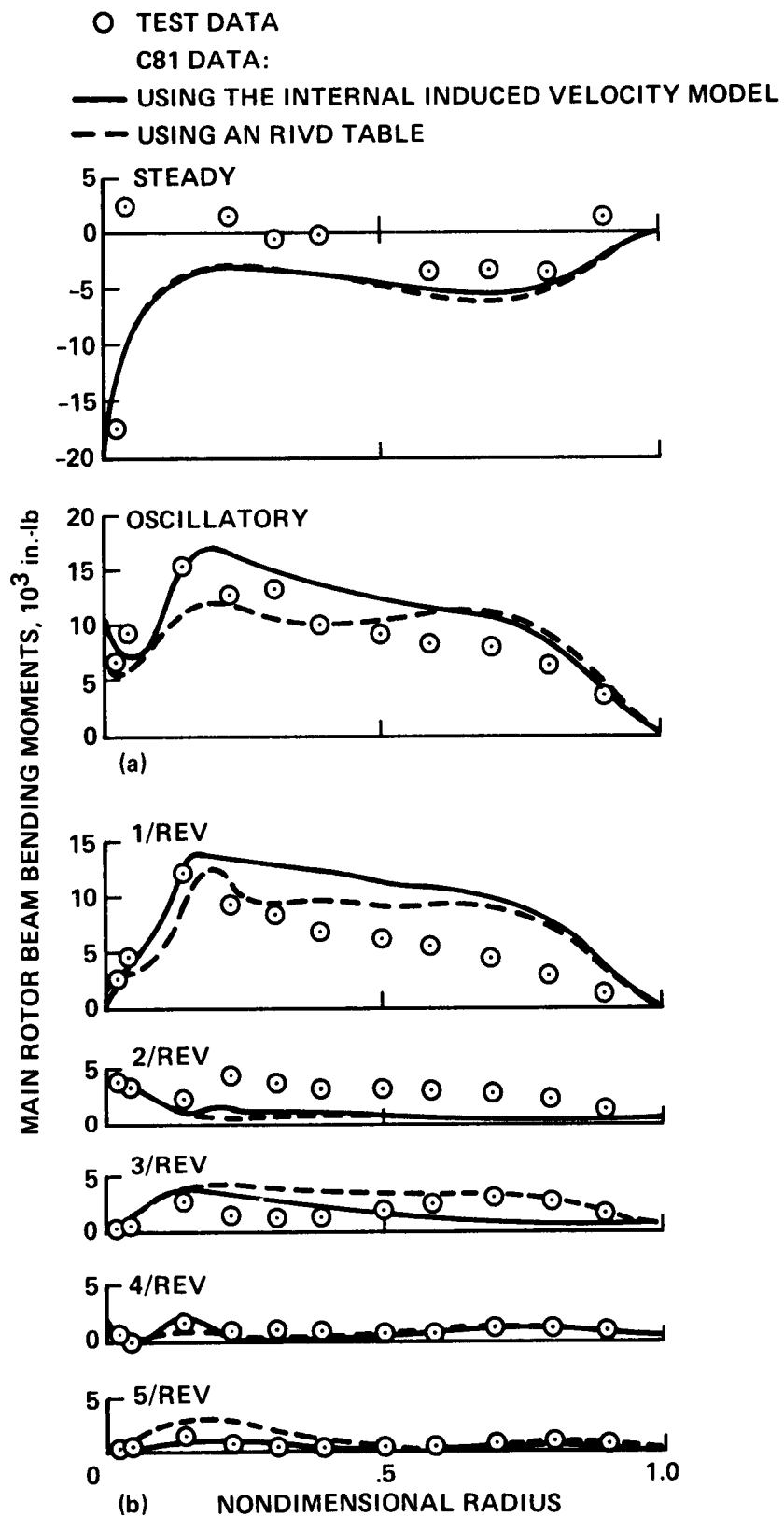


Figure 7: C81 results from reference 27.

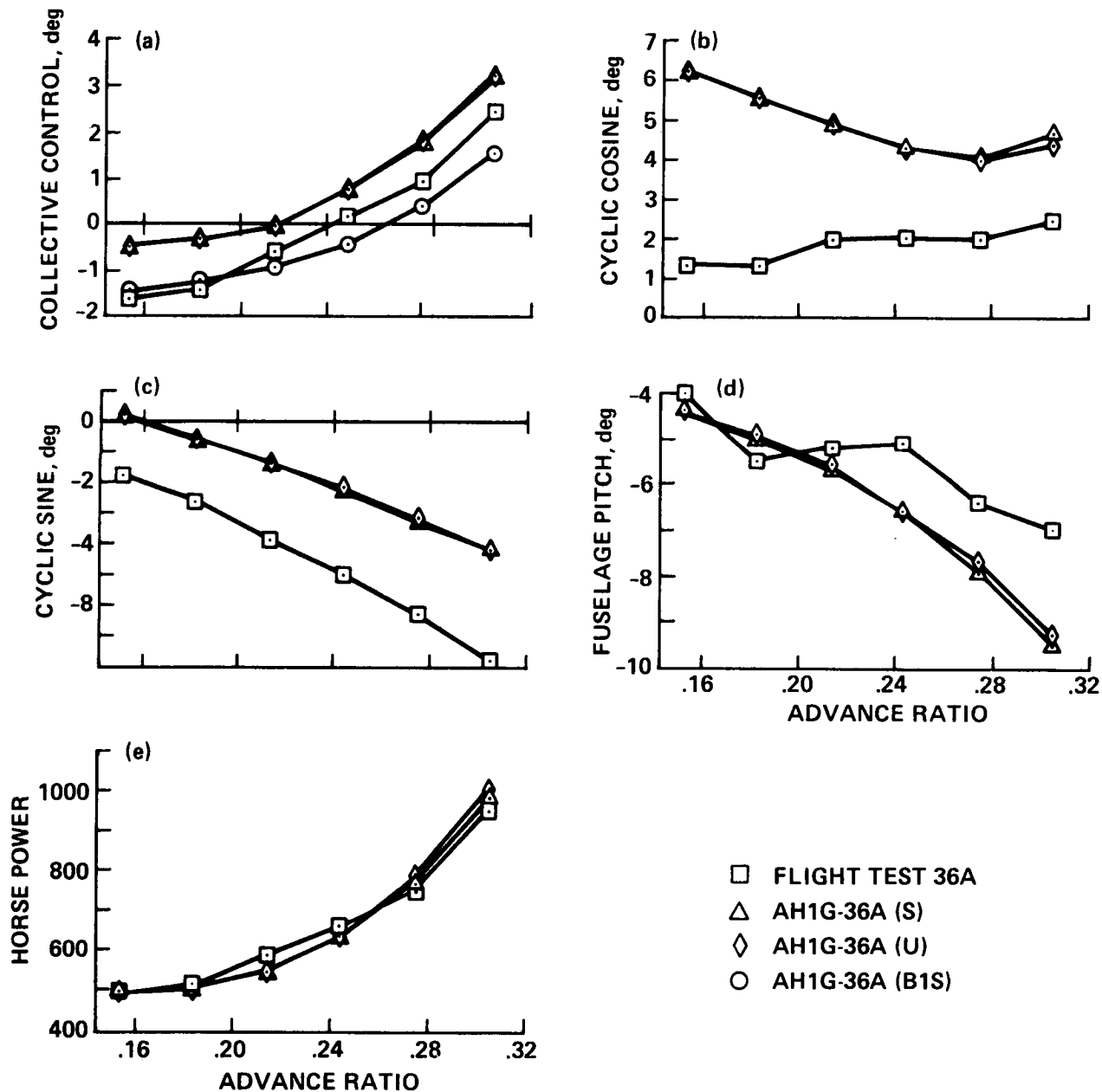


Figure 8: DYSCO results from reference 22.

accelerations, not from a rotor aerodynamic/dynamic analysis. A number of unvalidated applications are also provided in reference [5].

4.4 CAMRAD Correlation

The CAMRAD code has been used extensively in the past few years as an analytical tool in many investigations. These investigations are summarised in the following applications: hover loading calculations using prescribed wake geometry (ref. [34]), lateral flapping calculations using nonuniform inflow and free wake geometry (ref. [35]), influence of unsteady aerodynamics on hingeless rotor ground resonance (ref. [36]), XV-15 tiltrotor performance, loads, and stability (ref. [37]), XH-59 ABC performance and loads (ref. [38]), fully consistent coupling with unsteady aerodynamics finite-difference calculations of advancing tip transonic flow (ref. [39]), body-induced flow effects on rotors (ref. [40]), hingeless rotor ground-resonance stability in forward flight (ref. [41]), hingeless rotor performance and stability in hover (ref. [42]), advanced technology LHX rotor performance (ref. [43]), V-22 tiltrotor model whirl flutter stability (ref. [44]), performance, loads and stability calculations for design of a high speed tiltrotor (ref. [45]), and correlation with flight test measurements of trim, blade loads, and blade airloading (ref. [46]).

The CAMRAD code has obtained wide acceptance due to its comprehensive analysis capability and consistent mathematical basis. The code development uses prudent compromises in modeling capability and solutions in order to compute performance, loads, trim states, transients, and stability characteristics efficiently. It has been used as a testbed for examining the improvement possible when the analysis is coupled via file transfer with another program which calculates the unsteady rotor flow from a three dimensional, full potential, finite difference code from computational flow dynamics formulations (ref. [39]).

Results from reference [46] are used to indicate the predictive ability of CAMRAD when compared with the SA 349/2 helicopter flight-test data. In the report, the capability to model the aerodynamic behavior is given primary attention. The results selected for presentation here will emphasize the dynamic behavior. Figure 10 shows the results for the flatwise and edgewise bending moments and torsion moment for a blade where the rotor is subject to a high speed - low thrust flight condition. The results presented in reference [46] ranged over several blade stations with the analysis and flight test data correlating better as radial station moves inboard. The results in figure 10 are for a blade station close to midspan. The figure illustrates that the analytical results are converged at six modes. The flatwise moment distribution around the rotor azimuth gives good correlation with the test data; the edgewise moments are not well predicted on the advancing side of the rotor disk. This error is attributed to the com-

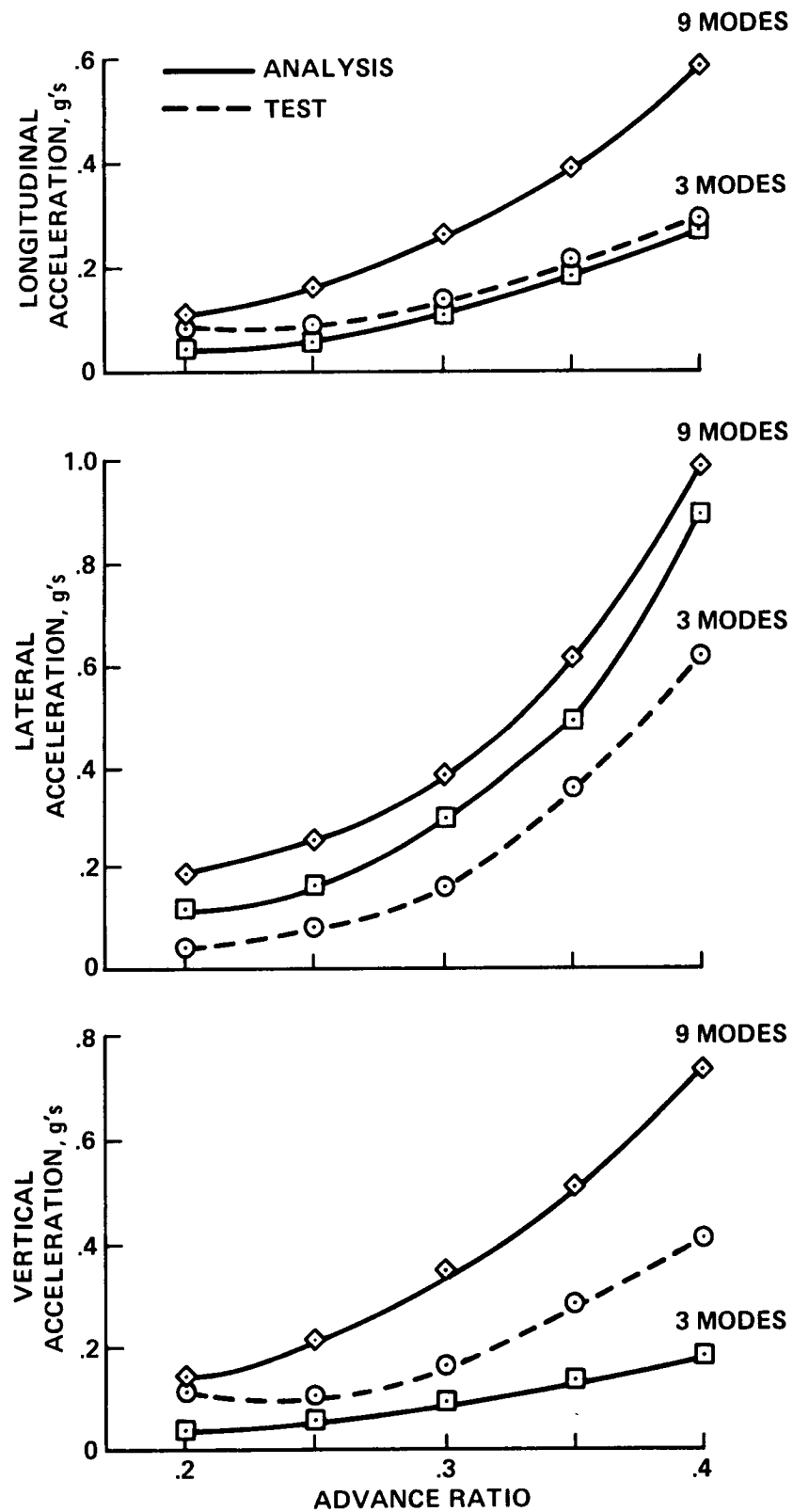


Figure 9: SIMVIB results from reference 5.

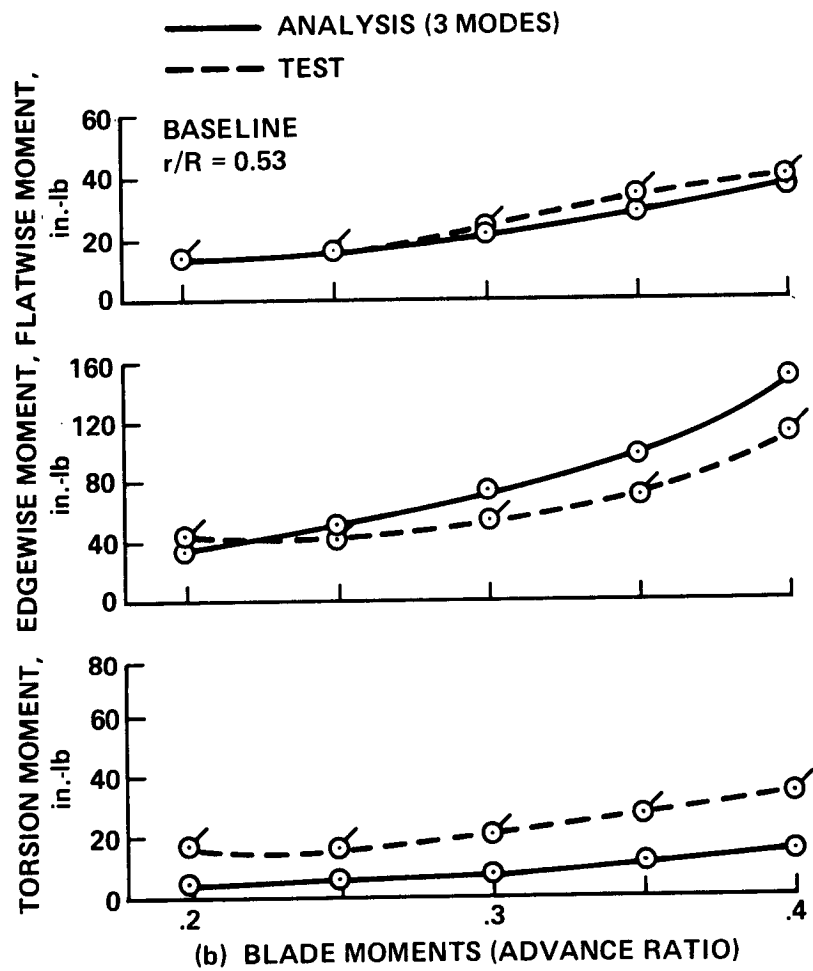
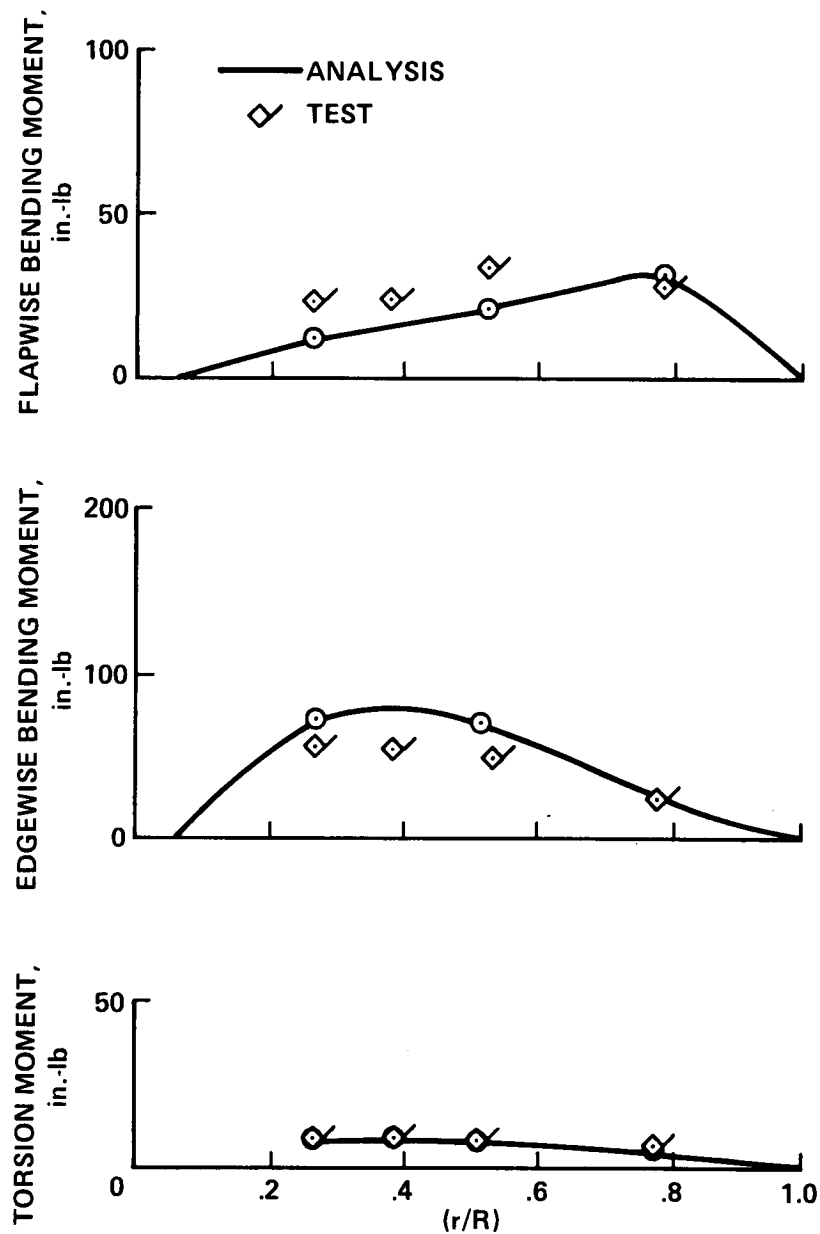


Figure 9: SIMVIB results - continued.



(c) BLADE MOMENTS (ADVANCE RATIO = 0.3)

Figure 9: SIMVIB results -- continued.

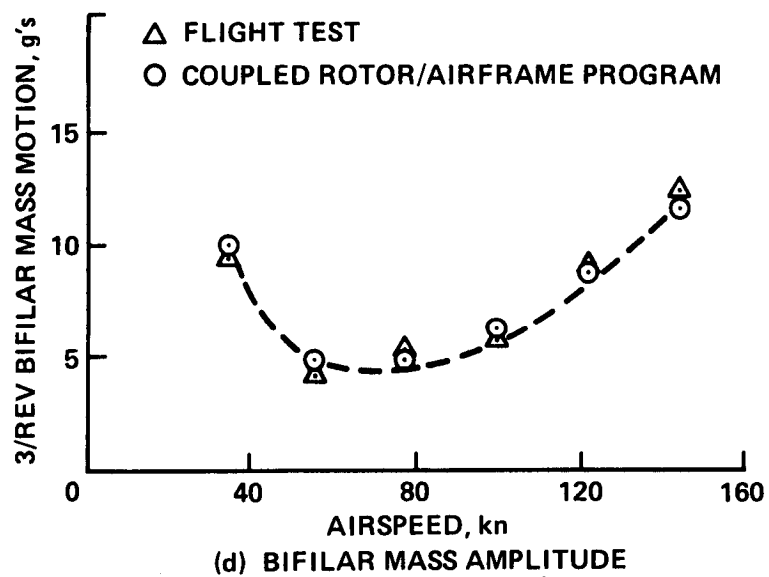


Figure 9: SIMVIB results – concluded.

compressibility effects not accounted for in the aerodynamic model. The torsional moment predictions are in reasonable agreement with test data. Convergence occurs with just two torsion modes required.

A low speed - low thrust flight condition case is also presented in reference [46]. Those results are duplicated here for variation of the coefficient of lift as a function of blade azimuth. At low speed the rotor wake stays in the vicinity of the blade and a strong blade-vortex interaction is obtained. A detailed wake geometry is needed to obtain good correlation for aerodynamic loads. As shown in figure 11, correlation to the wind tunnel data improves with each improvement to the wake geometry model for all blade stations. The uniform inflow analysis predicts the trends of lift variation around the rotor azimuth but not the details. The lift prediction is improved when the prescribed wake model is used. However, the free-wake model predicts the lift behavior very closely. It is in the area of aerodynamic modeling that CAMRAD excels. Even though lifting line theory is used, care has been taken to incorporate a free-wake geometry iteration with rollup and near and far wake effects included as well as corrections for blade tip, yawed flow, nonuniform inflow, dynamic stall, ground effect, and unsteady lift and moment.

4.5 GRASP Correlation

The GRASP code has been recently completed and the validation efforts are underway. Extensive comparisons with the flap-lag-torsion aeroelastic stability experimental results presented in reference [47] are in progress and will be reported in reference [48]. Reference [49] compares the GRASP solutions with theoretical results of Ormiston [50] for the ground and air resonance stability of a coupled rotor-fuselage. Good agreement was obtained for these results. Also in reference [49] the GRASP code is compared with a basic experiment carried out at Princeton University (refs. [51,52,53]). Typical results of this latter comparison are shown in figure 12. The beam is a slender, nonrotating, cantilevered, uniform beam with a tip mass. The load angle θ is varied from 0 to 90° at the beam root. These results show excellent agreement between the GRASP predictions and the experimental data for both the static deflection and the first flatwise and edgewise frequencies.

4.6 2GCHAS Schedule

The 2GCHAS code is under development. The first public release of the code is scheduled for the third quarter of 1988 as shown in figure 5. At that time, validation efforts will be initiated to qualify the accuracy of the integrated System.

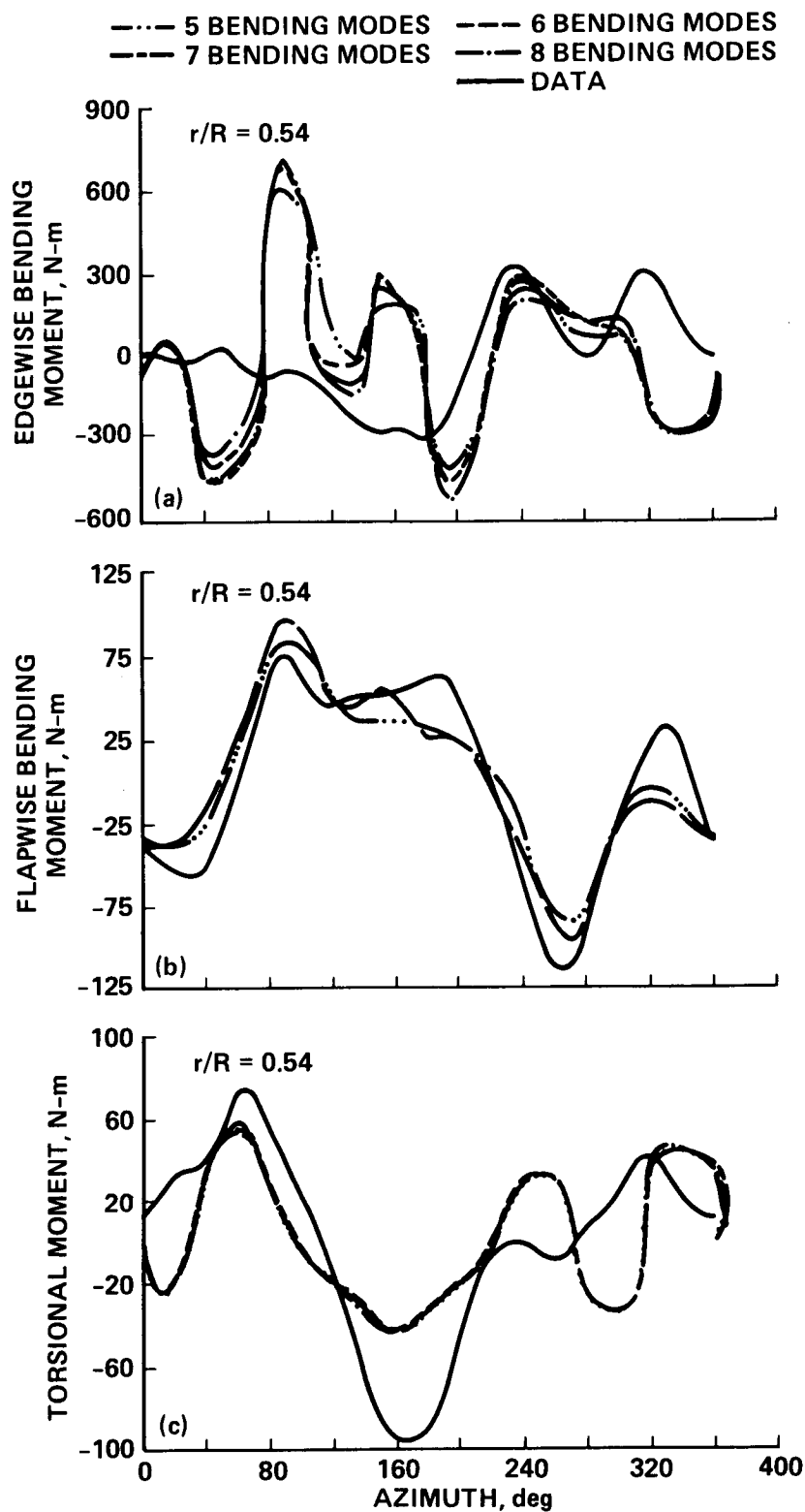


Figure 10: CAMRAD correlation from reference 46 for high speed ($\mu = .36$) and low thrust ($C_T/\sigma = 0.071$).

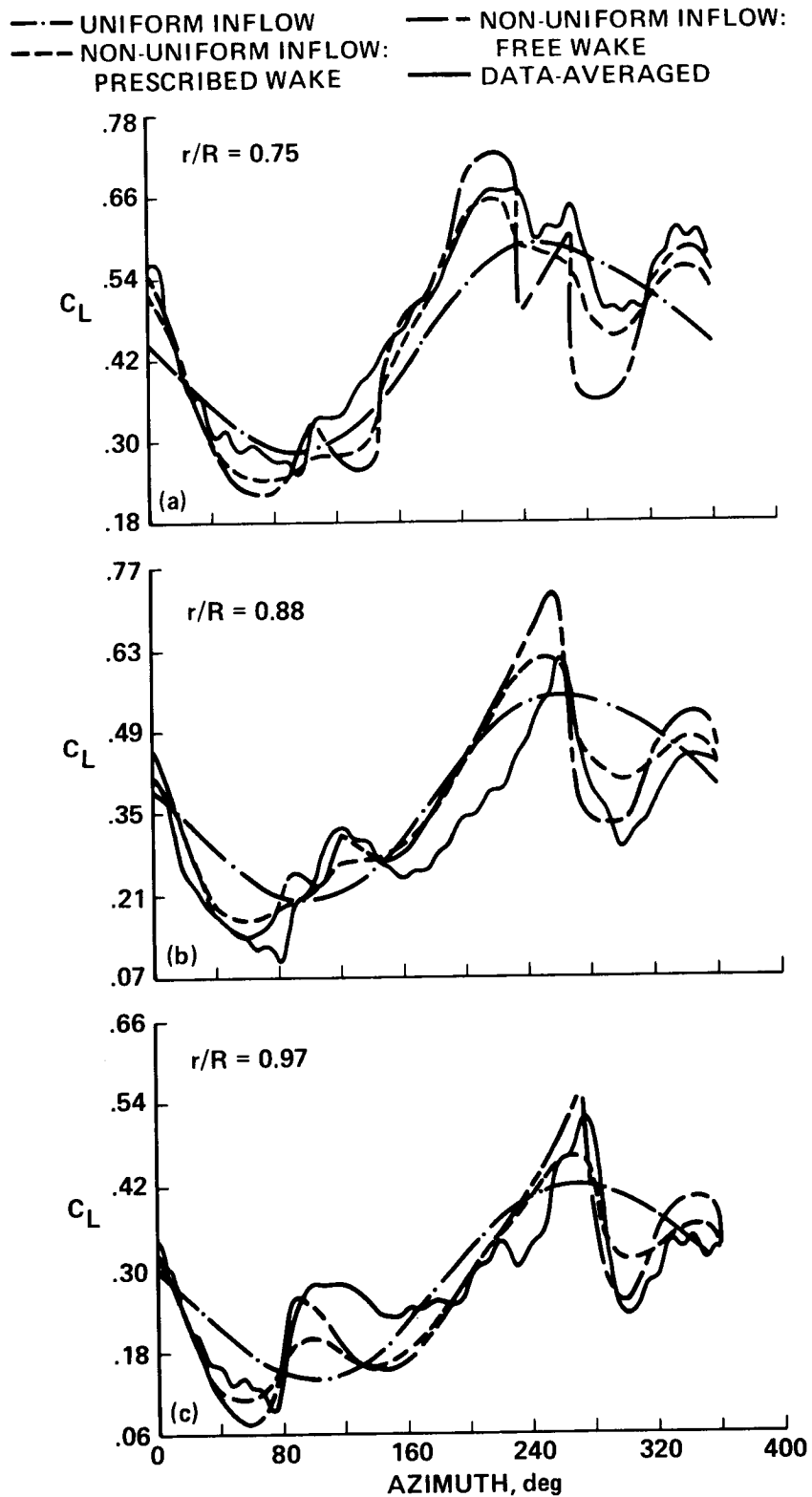


Figure 11: CAMRAD correlation from reference 46 for low speed ($\mu = 0.14$) and low thrust ($C_T/\sigma = 0.065$).

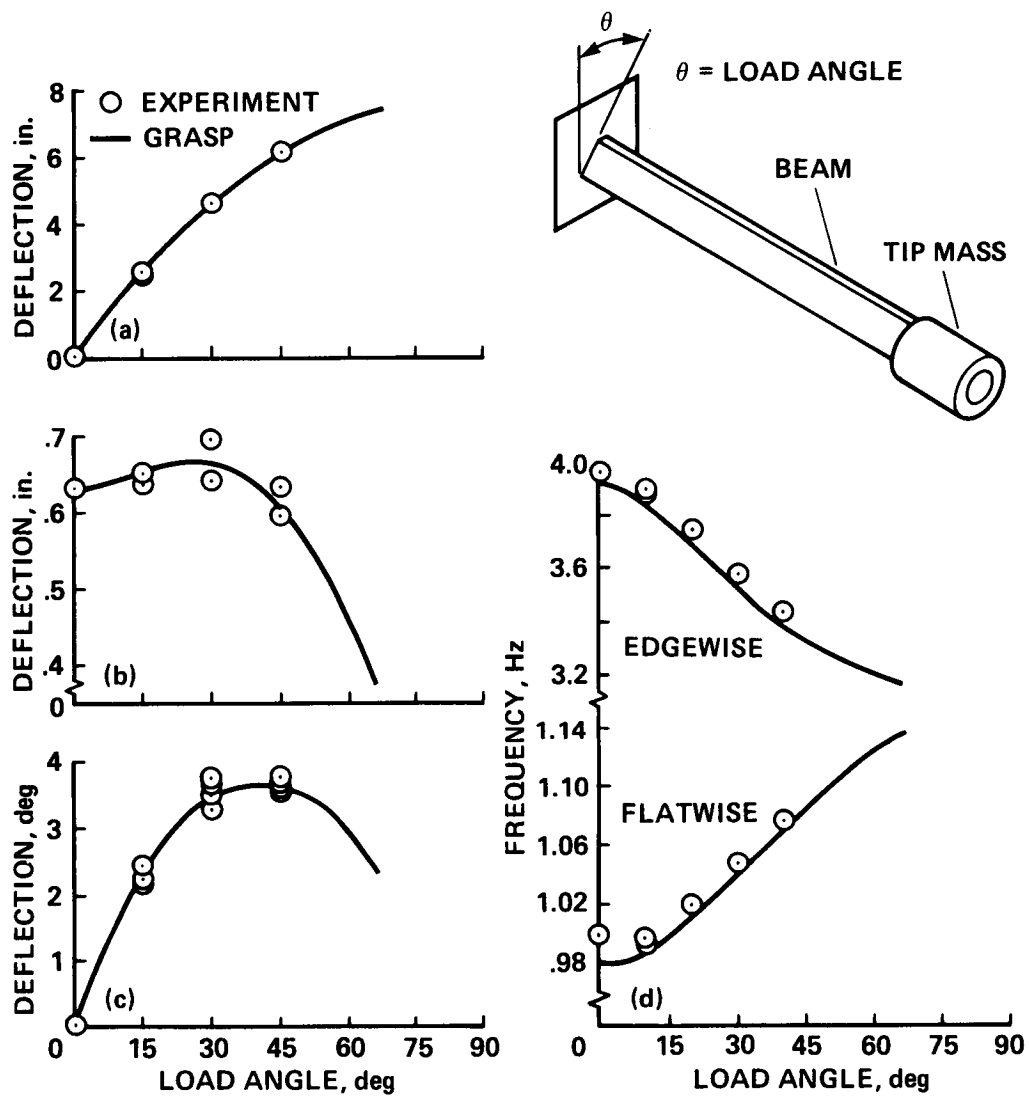


Figure 12: GRASP results from reference 49.

opment contractors will perform limited validation studies for their software contributions. The results of these initial validation studies will be available with the System document.

5 Concluding Remarks

The government's influence on interdisciplinary analysis software has been reviewed over the past decade. As a result of this involvement several significant advancements have been noted.

1. Both the government and the industry codes have become broader in scope and more consistent in the theoretical development. This is evident in the CAMRAD, GRASP and DYSCO codes presently and is anticipated for the 2GCHAS code.
2. New code development for interdisciplinary analysis requires an executive portion of the code to allow modular development and execution of the code subunits. 2GCHAS will have an extensive executive system. In addition, the other five codes discussed in this paper have addressed this feature to some extent. Even the oldest code surveyed, C81, recently underwent major modifications to allow a modular execution of its subsystems.
3. The trend in subunit development is to have all subprograms operate entirely in the executive environment. 2GCHAS, GRASP, DYSCO and CAMRAD are examples of this type of executive. C81 runs in an environment that appears to the user to be a single executive, but that executive actually runs up to eight separate job steps. The SIMVIB code uses a suite of independent programs which are coupled only by data files.
4. The structural modeling capability now exists to couple substructures. This capability is most easily incorporated in finite element based codes. The capability is best demonstrated in the GRASP code which combines finite element and multi-body coupling techniques. It is also demonstrated in the DYSCO and SIMVIB codes which include a linear Hurty coupling scheme for substructures. In these cases the subsystems being coupled do not have to be finite elements but must be consistent displacement-type math model formulations. The approach allows maximum flexibility in matching model details in the airframe and rotor system and computational efficiency when the finite element level of detail is not necessary. 2GCHAS will include the Hurty capability initially and then move to a more general approach including nonlinear coupling at a later date.

5. The inability to predict the rotorcraft aerodynamic environment and the time-varying aerodynamic loads on the rotors is perceived as the greatest impediment to good correlation between theory and test. Presently, aerodynamic modeling involves predicting the aerodynamic loads on a specific component with limited interference effects from other components accounted for. A general, interactional formulation is not available. There has been limited experience in interfacing advanced aerodynamics methodologies with comprehensive models. Comprehensive analysis methods with highly consistent dynamics components, reliable solution techniques and well documented interfaces to their aerodynamics analysis features should provide excellent test beds for advances in aerodynamics modeling. The mating of differencing schemes for unsteady aerodynamics of transonic tip flow to CAMRAD (ref. [39]) is an effort of this type. The 2GCHAS may provide additional impetus to generalizing aerodynamic approaches in software applications.
6. Validation of comprehensive codes is a massive, long term, and necessary undertaking. Comparisons with experimental data, other methods, closed form solutions and years of use in a production environment are necessary to build confidence in a design tool. The newer comprehensive codes will need continued government funding for validation studies to be performed to show both the areas acceptance and the areas where more rigorous development work must be done.
7. The modular component development and executive environment for multidiscipline analysis is in the infant stage and sufficient experience is not available to know how much of a boon this will be to research and to advancing the state of the art. However, it should at least be an asset to analysis and design problems and simplify and speed information and software transfer.
8. Wide distribution of government-sponsored software and theoretical developments should continue to be encouraged if the rotorcraft industry is to benefit fully from them. The importance of having widely accepted analysis tools available to industry and academia has been demonstrated by NASTRAN and other codes. Analyses and research have been conducted with these codes at manufacturing sites and universities. In addition to providing the immediate results for the research, the codes and their theoretical development have provided a basis for knowledgeable information exchanges. One form of the information exchange is to provide better prepared students from universities to the industry.
9. The government funding has been essential to the effort since the multidiscipline development requires broad areas of expertise and years of commitment.

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